IUHET 431
February 2001

TOPICS IN LORENTZ AND CPT VIOLATION[1]

V. Alan Kostelecký[2]

1. INTRODUCTION

Invariance under Lorentz and CPT transformations is a fundamental requirement of local relativistic quantum field theories, including the standard model of particle physics. This invariance also seems to be realized in nature, as no clear signals for violations have been observed despite numerous experimental tests. Nonetheless, several facts offer motivation for theoretical studies of possible Lorentz and CPT violation [1]. One is that quantitative statements about the degree to which nature exhibits Lorentz and CPT symmetry are best expressed within a consistent and general theoretical framework that allows for violations [2, 3, 4]. Another more subtle fact is that the exceptional sensitivity of present experimental tests [5] implies access to highly suppressed Lorentz and CPT violations that might arise at scales well beyond the standard model in the context of Planck-scale physics [6].

At earlier conferences in this series [7] I have discussed the possibility that Lorentz and CPT symmetry might be violated as a result of new physics in a theory underlying the standard model, perhaps including string theory [6]. I have also described the Lorentz- and CPT-violating standard-model extension that allows for the associated low-energy effects in a very general context [2, 3] and outlined some of the numerous existing and future experiments that test these ideas. These experiments include, for example, studies of neutral-meson oscillations [8, 2, 9, 10, 11, 12], comparative tests of QED in Penning traps [13, 14, 15, 16], spectroscopy of hydrogen and antihydrogen [17, 18], measurements of muon properties [19, 20], clock-comparison experiments [21, 22, 23, 24], observations of the behavior of a spin-polarized torsion pendulum [25, 26], measurements of cosmological birefringence [27, 3, 28, 29], and observations of the baryon asymmetry [30].

In the present talk I briefly describe some of our recent theoretical analyses, emphasizing in particular topics at the level of quantum field theory [2, 3] and the issues of causality and stability in Lorentz-violating theories [4]. Other studies of Lorentz and CPT violation in the context of the standard-model extension are being presented at this conference [33, 34]. The reader may also find of interest recent

[1]Presented at Orbis Scientiae 2000, Fort Lauderdale, Florida, December 2000

[2]Physics Department, Indiana University, Bloomington, IN 47405, U.S.A.

efforts to create a classical analogue for CPT violation [31, 32], which lie outside the scope of this talk. A treatment of experimental results released in the year since the previous conference in this series is also outside the present scope. I mention here only the newly published results on Lorentz and CPT violation involving protons [18] and neutrons [24], and the preliminary announcement of results in the muon sector obtained from muonium hyperfine spectroscopy [20].

2. CONCEPTUAL BASICS

Over the past decade, a framework allowing for Lorentz and CPT violation within realistic field-theoretic models has been developed [6] that leads to a phenomenology for Lorentz and CPT violation at the level of the standard model and quantum electrodynamics (QED) [2]. The resulting general standard-model extension [3] can be chosen to preserve the usual SU(3)$\times$SU(2)$\times$U(1) gauge structure and to be power-counting renormalizable. Energy and momentum are conserved, and conventional canonical methods for quantization apply. In this part of the talk, I summarize some useful basic concepts for the theoretical results to follow.

Observer and particle Lorentz transformations. By construction, the standard-model extension is invariant under rotations or boosts of an observer's inertial frame, called observer Lorentz transformations. These must be contrasted with rotations or boosts of the localized fields in a fixed observer coordinate system, called particle Lorentz transformations, which can change the physics [3]. The observer Lorentz invariance of the standard-model extension, together with its generality, means that it is the low-energy limit of *any* realistic underlying theory in which the physics is coordinate independent in an inertial frame but in which Lorentz symmetry is broken. Lorentz- and CPT-violating effects could therefore provide a unique low-energy signature for qualitatively new physics from the Planck scale.

One attractive scenario for generating Lorentz- and CPT-violating terms while maintaining observer Lorentz invariance is to invoke spontaneous Lorentz breaking in an underlying fully Lorentz-covariant theory at the Planck scale, perhaps string theory [6]. Since observer Lorentz invariance constrains the physical behavior under suitable coordinate changes made by an external observer, an underlying theory incorporating this property cannot lose it through internal interactions such as those leading to spontaneous Lorentz violation. If instead the underlying theory were to break Lorentz invariance explicitly, then observer Lorentz invariance would appear unnatural and imposing it would involve an extra requirement.

Stability and causality. Two crucial features of acceptable physical quantum field theories are stability and causality. In relativistic field theories, these are closely linked to Lorentz invariance [35]. Among their implications are the requirements of energy positivity at arbitrary momenta and of commutativity of spacelike-separated observables, both of which must hold in all observer inertial frames. For the standard-model extension, which allows for (particle) Lorentz violation, it is natural to ask about the implications of these requirements both within the theory and in the larger context of the underlying Planck-scale physics [4].

In addressing many aspects of this question, the complications of the full standard-model extension can be avoided by limiting attention to the case of the quadratic fermion part of a general renormalizable lagrangian with explicit Lorentz-

and CPT-breaking terms. This is the single-fermion limit of the free-matter sector in the general standard-model extension, given by [3]:

$$\mathcal{L} = \tfrac{1}{2} i \overline{\psi} \Gamma^\nu \overleftrightarrow{\partial}_\nu \psi - \overline{\psi} M \psi, \tag{1}$$

where

$$\Gamma^\nu \equiv \gamma^\nu + c^{\mu\nu}\gamma_\mu + d^{\mu\nu}\gamma_5\gamma_\mu + e^\nu + i f^\nu \gamma_5 + \tfrac{1}{2} g^{\lambda\mu\nu}\sigma_{\lambda\mu} \tag{2}$$

and

$$M \equiv m + a_\mu \gamma^\mu + b_\mu \gamma_5 \gamma^\mu + \tfrac{1}{2} H^{\mu\nu}\sigma_{\mu\nu}. \tag{3}$$

The parameters a_μ, b_μ, $c_{\mu\nu}$, ..., $H_{\mu\nu}$ control the degree of Lorentz and CPT violation, and their properties are discussed in Ref. [3]. Note that observer Lorentz symmetry is manifest in this model because the lagrangian (1) is independent of the choice of coordinate system. However, particle Lorentz transformations modify the fields but leave invariant the coefficients a_μ, b_μ, ..., $H_{\mu\nu}$, thereby breaking Lorentz symmetry.

A satisfactory investigation of stability and causality for Eq. (1) or the standard-model extension must incorporate the implications of observer Lorentz invariance [4]. For example, if energy positivity is found to be violated for specified coefficients for Lorentz violation in some inertial frame, then a corresponding difficulty must be present in every other inertial frame. Conversely, if complete consistency can be established in any inertial frame, then observer Lorentz invariance ensures consistency in all other inertial frames.

For simplicity and definiteness in what follows, I take the mass m of the fermion in Eq. (1) to be nonzero. This is certainly appropriate for the non-neutrino fermionic sector of the standard-model extension, and also applies if neutrinos have mass with possible minor modifications for Majorana fermions. Many of the results obtained can also be applied to bosons and to the massless case, although some care would be required to handle correctly the additional complications arising from distinctions between finite- and zero-mass representations of the Lorentz group. This is particularly true for gauge bosons [4], for which a satisfactory analysis of causality and stability remains an open problem. Some results about causality restricted to the special case of a single Lorentz-violating term in the photon sector have recently been obtained [36].

Concordant frames. The coefficients for Lorentz violation a_μ, b_μ, $c_{\mu\nu}$, ..., $H_{\mu\nu}$ in Eq. (1) transform as nontrivial representations of the observer Lorentz group O(3,1). Since this group is noncompact, individual components of these coefficients can become arbitrarily large. Under certain circumstances, including both exact and perturbative calculations, it is therefore valuable to define a special class of inertial frames called concordant frames, in which the coefficients for Lorentz and CPT violation represent only a small perturbation relative to the ordinary Dirac case [4].

Since no Lorentz and CPT violation has been observed in nature, any effects are presumably minuscule in an Earth-based laboratory. Barring unexpected surprises such as fortuitous cancellations, this suggests that all the coefficients in Eq. (2) are well below 1 and those in Eq. (3) are well below m. It can then be regarded as an experimental fact that any inertial frame in which the Earth moves nonrelativistically is a concordant frame.

If small Lorentz- and CPT-violating effects in nature indeed arise from an underlying theory at some large scale M_P, such as the Planck scale, then the natural dimensionless suppression factor is some power of the ratio m/M_P [2]. The size of the coefficients $c_{\mu\nu}$, $d_{\mu\nu}$, e_μ, f_μ, $g_{\lambda\mu\nu}$ in Eq. (2) is therefore likely to be no larger than m/M_P, although smaller values are possible. Similarly, the size of the coefficients a_μ, b_μ, $H_{\mu\nu}$ in Eq. (3) is likely to be no larger than m^2/M_P.

High-energy physics. Within standard special relativity, the separation between high- and low-energy physics is frame independent. Thus, high-energy physics in one inertial frame corresponds to high-energy physics in another frame. However, this correspondence fails in the presence of Lorentz violation [4]. The point is that the coefficients for Lorentz and CPT violation determining the physics of a high-energy particle in one inertial frame can be very different from those determining the high-energy physics in another frame. The breaking of particle Lorentz invariance therefore implies that high-energy physics can change between inertial frames, despite the observer Lorentz invariance. In particular, statements concerning Lorentz-breaking effects restricted to high energies may be observer dependent.

Since the standard concept of high and low energy is ambiguous, a cleaner definition is useful [4]. A useful option is to take the separation between high and low energies relative to the scale of the underlying theory as being defined in a concordant frame. This definition is experimentally reasonable and compatible with intuition and common usage, since any laboratory frame moves nonrelativistically relative to a concordant frame, so high- and low-energy physics are similar in both.

3. QUANTUM MECHANICS AND QUANTUM FIELD THEORY

In a concordant frame, where the coefficients for Lorentz violation are small, the usual methods of relativistic quantum mechanics and quantum field theory can be adopted. The first step is the construction of the relativistic quantum hamiltonian H from the lagrangian $\mathcal{L}$ of Eq. (1). Care is required because $\mathcal{L}$ contains extra time-derivative terms. A spinor redefinition can be used to eliminate these couplings in a concordant frame [13]: define $\psi = A\chi$ and require the nonsingular spacetime-independent matrix A to obey $A^\dagger \gamma^0 \Gamma^0 A = I$. The reader is warned that the explicit form of A depends on the chosen inertial frame. In any case, it follows that $\mathcal{L}[\chi]$ contains no time derivatives other than the usual one, $\frac{1}{2} i \overline{\chi} \gamma^0 \overleftrightarrow{\partial_0} \chi$. Since the conversion of ψ to χ can be regarded as a change of basis in spinor space, the physics is unaffected.

It is known that A exists if and only if all the eigenvalues of $\gamma^0\Gamma^0$ are positive. Quantitatively, a parameter δ^0 can be defined as the upper bound on the size of certain coefficients for Lorentz and CPT violation such that A exists [4]. It can be proved that $\delta^0 < 1/480$, which represents a value far larger than the maximum size of δ^0 likely to be acceptable on experimental grounds. The spinor redefinition involving A therefore always exists in a concordant frame and is applicable to realistic situations in nature.

After implementing the spinor redefinition from ψ to χ, one can use the Euler-Lagrange equations to obtain a modified Dirac equation in terms of χ. This takes the form

$$(i\partial_0 - H)\chi = 0, \tag{4}$$

where the hamiltonian $H = A^\dagger \gamma^0 (i\Gamma^j \partial_j - M) A$ is hermitian. Various explicit forms for this hamiltonian can be found in Ref. [22].

The modified Dirac equation (4) is solved via a superposition of plane spinor waves, as usual: $\chi(x) = w(\lambda) \exp(-i\lambda_\mu x^\mu)$. The quantity λ_μ obeys the dispersion relation

$$\det(\Gamma^\mu \lambda_\mu - M) = 0. \tag{5}$$

This dispersion relation is displayed as an explicit polynomial in Ref. [4]. It can be regarded as a quartic equation for $\lambda^0(\vec{\lambda})$. For a particle with definite 3-momentum, the dispersion relation fixes the exact eigenenergies in the presence of Lorentz and CPT violation. All four roots of Eq. (5) are necessarily real, since H is hermitian in a concordant frame. They are also independent of the spinor redefinition. The dispersion relation is observer Lorentz invariant, so λ_μ must be an observer Lorentz 4-vector.

In the usual Dirac case, the roots of the dispersion relation exhibit a fourfold degeneracy. However, in the presence of Lorentz and CPT violation this degeneracy is typically lifted. Nonetheless, for sufficiently small Lorentz and CPT violation, the roots can still be separated into two positive ones and two negative ones. The criterion for the existence of this separation can be quantitatively expressed in terms of a parameter δ, defined in Ref. [4]. It is known that the bound $\delta < m/124$ is sufficient. The values of this bound is again much larger than experimental observations are likely to allow, so the existence of Lorentz and CPT violation in nature would have no effect on the separation between positive and negative roots. Note that this bound is independent of the spinor redefinition. The two bounds on δ^0 and δ provide criteria quantitatively constraining the definition of a concordant frame.

As usual, the eigenfunctions of the modified Dirac equation (4) corresponding to the two negative roots can be reinterpreted as positive-energy reversed-momentum wave functions. The four resulting spinors u and v are eigenvectors of the hermitian hamiltonian H. They span the spinor space and can be used to write a general solution of Eq. (4) as a Fourier superposition of plane wave solutions with complex weights in the standard way.

To convert from relativistic quantum mechanics to quantum field theory, the complex weights in the Fourier expansion are promoted to creation and annihilation operators on a Fock space, as usual. The spinors ψ and χ become quantum fields, related through the redefinition $\psi = A\chi$ as before. The standard nonvanishing anticommutation relations can be imposed on the creation and annihilation operators. The resulting equal-time anticommutators for the fields χ are conventional, while the nonvanishing equal-time anticommutators for the original fields ψ become [4]

$$\{\psi_j(t, \vec{x}), \overline{\psi}_l(t, \vec{x}\,')\Gamma^0_{lk}\} = \delta_{jk}\delta^3(\vec{x} - \vec{x}\,'), \tag{6}$$

where the spinor indices are explicitly shown. Note the generalization from the usual Dirac case of the canonical conjugate of ψ, which in the presence of Lorentz and CPT violation takes the form $\pi_\psi = \overline{\psi}\Gamma^0$, with Γ^0 given by Eq. (2).

In a concordant frame, the vacuum state $|0\rangle$ of the Hilbert space is defined as the state that vanishes when the annihilation operators are applied. The creation operators then act on $|0\rangle$ to yield states describing particles and antiparticles with 4-momenta appropriately determined by the dispersion relation (5). An important

consequence is that the zero components of these 4-vectors are positive definite. This implies positivity of the energy for the Hilbert-space states in the concordant frame [4].

4. STABILITY AND CAUSALITY

The derivation of the quantum physics associated with the lagrangian (1) can be performed as outlined in the previous section provided the constraints on δ^0 and δ are satisfied. These constraints ensure that the Lorentz-violating time-derivative terms can be eliminated and that the usual separation holds between particles and antiparticles, and they quantify the notion of concordant frame. They involve specific components of the parameters for Lorentz and CPT violation and so are noninvariant under observer Lorentz transformations, as expected. A class of observers therefore exists for whom these bounds are violated and the derivation of the quantum physics fails. These observers are strongly boosted relative to a concordant frame. Nonetheless, when combined with the requirement of observer Lorentz invariance, their existence indicates some difficulty must occur even for the quantization scheme in a concordant frame. These are associated with the stability and causality of the theory, which are considered next.

Stability. In conventional relativistic field theory with Lorentz symmetry, energy positivity in a given frame implies that the vacuum is stable in any frame provided certain conditions are met. One is that for all one-particle states in the given frame the 4-momenta are timelike or lightlike with nonnegative zeroth components. Since the signs of these zeroth components are invariant under an observer Lorentz transformation, energy positivity is a Lorentz-invariant concept despite being a statement about a 4-vector component. Thus, for example, the usual free Dirac theory exhibits energy positivity in all observer frames.

In contrast, for a theory with Lorentz and CPT violation, results demonstrating energy positivity in one frame are insufficient to ensure energy positivity or stability in all inertial frames. This is true, for example, of the energy positivity discussed in the previous section for the theory (1) in a concordant frame. At least one of the usual assumptions fails: certain energy-momentum 4-vectors satisfying the dispersion relation (5) may in fact be spacelike in all observer frames. An example is provided by the dispersion relation for the special case of the theory (1) in which only the b_μ coefficient is nonzero. No matter how small b_μ is chosen, an observer frame can always be found in which spacelike 4-vectors λ^μ exist that satisfy the dispersion relation [4].

Observer Lorentz invariance ensures that the instabilities due to the existence of spacelike solutions exist in any frame, including concordant ones. However, they are perhaps most intuitively appreciated by considering an appropriate observer boost. With a boost velocity less than 1, it is always possible to convert a spacelike vector with a positive zeroth component to one with a negative zeroth component. This guarantees the existence of a class of observer frames for which a single root of the dispersion relation involves both positive and negative energies, and it implies the canonical quantization procedure fails.

It is of interest to determine the scale $\tilde{M}$ of the 3-momentum at which the 4-momentum turns spacelike. As an example, consider the special case of the the-

ory (1) with only a nonzero timelike b_μ. In the observer frame with $b_\mu = (b_0, \vec{0})$, and taking $b_0 \sim \mathcal{O}(m^2/M_P)$ according to the discussion of scales in section 2, it follows that $\widetilde{M} \gtrsim \mathcal{O}(M_P)$. This shows that instabilities arise only for Planck-scale 4-momenta in any of the concordant frames. The associated negative-energy problem in boosted frames emerges only for observers undergoing Planck-scale boosts. The concordant-frame quantization described in section 3 therefore maintains stability for all experimentally attainable physical momenta and in all experimentally attainable observer frames.

The existence of observer Lorentz invariance implies that the negative-energy instabilities in strongly boosted frames must have a counterpart in concordant frames, albeit restricted to particles with Planck-scale energies. This is indeed the case: single-particle states with Planck-scale energies in a concordant frame are unstable to decay. For example, a Planck-energy fermion can explicitly be shown to be unstable to the emission of a fermion-antifermion pair [4]. In conventional QED, this process is kinematically forbidden. However, the presence of spacelike momenta in the context of the Lorentz- and CPT-violating QED extension makes the emission proceed at Planck energies. Other conventionally forbidden processes are also likely to occur. A single-particle state describing a fermion of sufficiently large 3-momentum is therefore unstable even in a concordant frame.

Causality. Microcausality holds in a quantum field theory provided any two local observables with spacelike separation commute. In a theory of Dirac fermions, the local quantum observables are fermion bilinears. Microcausality therefore holds for the modified theory (1) if

$$iS(x - x') = \{\psi(x), \overline{\psi}(x')\} = 0, \quad (x - x')^2 < 0 \tag{7}$$

is satisfied. Note that the original field ψ is involved, rather than the redefined field χ, because the definition of the latter depends on the choice of inertial frame.

An integral representation for $S(x - x')$ provides a useful tool in studying the conditions for microcausality. In a concordant frame, an integral represention for the anticommutator function $S(x - x')$ can be obtained as [4]

$$S(z) = \mathrm{cof}(\Gamma^\mu i\partial_\mu - M) \int_C \frac{d^4\lambda}{(2\pi)^4} \frac{e^{-i\lambda\cdot z}}{\det(\Gamma^\mu \lambda_\mu - M)}. \tag{8}$$

Provided $c_{\mu\nu} = d_{\mu\nu} = e_\mu = f_\mu = g_{\lambda\mu\nu} = 0$, the derivative couplings take the standard form with $\Gamma^\mu = \gamma^\mu$. In this case, a hermitian hamiltonian always exists and the four poles remain on the real axis. Explicit calculation of the contour integration in Eq. (8) shows that $S(z)$ vanishes outside the light cone in this case. It follows that the theory (1) restricted to nonzero a_μ, b_μ and $H_{\mu\nu}$ is microcausal. However, in the unrestricted theory (1), the poles of the integrand in Eq. (8) can lie away from the real λ^0 axis, in which case the contour C may fail to encircle them. This difficulty occurs when the bound on δ^0 discussed in section 3 is violated. The hamiltonian then cannot be made hermitian, and the roots of the dispersion relation may be complex.

In discussions of causality, it is advantageous to define the velocity of a particle at arbitrary 3-momentum. However, the definition of the quantum velocity operator is nontrivial even for the Lorentz- and CPT-invariant case and becomes involved when these symmetries are violated [3]. One useful concept is the group

velocity, which can be defined for a monochromatic wave in terms of the dispersion relation by $\vec{v}_g = \partial E/\partial \vec{p}$, as usual. It can be shown that the flow velocities of the conserved momentum and the conserved charge for one-particle states agree with the group velocity. Also, explicit checks in special cases suggest that $\langle d\vec{x}/dt \rangle = \vec{v}_g$ in relativistic quantum mechanics, and that the maximal group velocity attainable has magnitude equal to the maximal signal speed obtained from $S(z)$.

The group velocity can be used to establish the scale $\tilde{M}$ of microcausality violation. Setting the group velocity to 1 and solving for the magnitude $|\vec{p}|$ of the 3-momentum determines the scale $\tilde{M}$ through $\tilde{M} = |\vec{p}|$. For example, consider the theory (1) with only the term e_μ nonzero [4]. Suppose e_μ is timelike, and take $\vec{e} = 0$ in the chosen concordant frame. Then, assuming the maximal expected Lorentz violation $e_0 \sim \mathcal{O}(m/M_P)$ following the discussion in section 2, the scale $\tilde{M}$ of microcausality violation is obtained as $\tilde{M} \gtrsim \mathcal{O}(M_P)$. Thus, the e_μ model violates microcausality at the scale M_P.

Intuition about the connection between hermiticity of the hamiltonian H and microcausality can also be obtained from the theory involving only e_μ. In this case, the nonzero entries of the matrix $\gamma^0\Gamma^0$ in the Pauli-Dirac representation consist only of diagonal entries $1 \pm e_0$. For $|e_0| < 1$, the spectrum of $\gamma^0\Gamma^0$ is therefore positive, and both a suitable matrix A and a hermitian hamiltonian H exist. However, when $|e_0| > 1$ the spectrum of $\gamma^0\Gamma^0$ includes two negative eigenvalues, and neither A nor a hermitian H exists. In the dispersion relation, the associated difficulty is that for $|e_0| > 1$ it is always possible to find an observer frame in which the roots become complex.

Intermediate-scale physics. The above results indicate that stability and causality violations emerge at a scale $\mathcal{O}(M_P)$. However, this conclusion may fail for the special case of theories of the form (1) with a nonzero coefficient $c_{\mu\nu}$ [4]. The point is that field operators with closely related derivative and spinor structures are involved for both the usual Dirac kinetic term and the term with coefficient $c_{\mu\nu}$, which means the latter behaves in many respects as a first-order correction to a zeroth-order result. This feature is unique to the $c_{\mu\nu}$ term.

To illustrate this point with a definite example, consider the special case of the lagrangian (1) with only the coefficient c_{00} nonzero in a concordant frame, and suppose in accordance with the discussion in section 2 that $c_{00} \sim \mathcal{O}(m/M_P)$. The dispersion relation for this model in an arbitrary frame is

$$(\eta_{\alpha\mu} + c_{\alpha\mu})(\eta^{\alpha}{}_{\nu} + c^{\alpha}{}_{\nu})\lambda^\mu\lambda^\nu - m^2 = 0. \tag{9}$$

For the case $c_{00} > 0$ it can then be shown that spacelike 4-momenta occur at a scale $\tilde{M} \gtrsim \mathcal{O}(\sqrt{mM_P})$, and so instabilities occur at energies well below the scale M_P of the underlying theory in this case. If instead $c_{00} < 0$, then it can be shown that at the same scale microcausality violations arise instead: the integration in (8) can be performed analytically, revealing that the anticommutator function $S(z)$ can be nonzero outside the region defined by $z^0 < (1 + c_{00})|\vec{z}|$ and that signal propagation could therefore occur with maximal speed $1/(1 + c_{00}) > 1$.

These results concerning intermediate scales are of interest because energies comparable to the order of $\sqrt{mM_P}$ in the concordant frame are exhibited by certain physical phenomena. For instance, it has been suggested that effects from c_{00}-type terms might be responsible for the apparent excess of cosmic rays in the region of 10^{19} GeV [37, 38]. The above analysis suggests that these effects can be traced

to stability or causality violations, which surely must be absent in a satisfactory underlying theory. It therefore seems plausible that the effective dispersion relation involving the coefficients $c_{\mu\nu}$ is modified already at these scales, perhaps along the lines described in the next section, but in any case in a way that preserves stability and causality. A corresponding modification of the predictions for cosmic rays and other phenomena at the scale $\sqrt{mM_P}$ would then be likely.

5. PLANCK-SCALE EFFECTS

The results described in section 4 indicate that problems with stability and causality in the theory (1) arise primarily for Planck-scale 4-momenta in a concordant frame or for observers undergoing a Planck boost relative to this frame. Although perhaps strictly unnecessary from the phenomenological viewpoint of current laboratory physics, it would be of interest to establish a framework for Lorentz and CPT violation in which both stability and microcausality hold.

A natural question is whether spontaneous Lorentz and CPT breaking suffices to avoid the issues with stability and causality. Since this type of breaking could occur in a theory with a Lorentz-invariant lagrangian and hence with Lorentz-covariant dynamics, it is likely to avoid at least some of the problems faced by theories with explicit Lorentz and CPT violation. For example, one important advantage of spontaneous Lorentz violation is the natural occurrence of observer Lorentz invariance, which eliminates the coordinate dependence and related problems faced by theories with explicit violation. However, spontaneous Lorentz violation manifests itself physically because the Fock-space states are constructed on a noninvariant vacuum, whereas stability and causality in a relativistic theory depend partly on the existence of a Lorentz-invariant vacuum. It is therefore to be expected that, despite the advantages of spontaneous Lorentz violation, some difficulties with stability and causality remain.

This expectation can be directly confirmed by studying the fermion sector of quantum field theories with spontaneous Lorentz violation [4]. The key point is that the lagrangian (1) includes by construction the most general terms quadratic in fermion fields that appear in a renormalizable theory. This means that implementing spontaneous Lorentz and CPT violation in any conventional fermion field theory must result in free-fermion Fock-space states with dispersion relations described by Eq. (5) or a restriction of it. If indeed all possible dispersion relations have either stability or causality violations at some large scale, then no fermion lagrangian with spontaneous Lorentz and CPT violation can have a completely satisfactory perturbative Hilbert space in conventional quantum field theory. Maintaining stability and causality would therefore require an additional ingredient beyond conventional quantum field theory, in accordance with the notion that Lorentz and CPT violation is a unique potential signal for Planck-scale physics.

The above discussion suggests that a theory with a quadratic lagrangian that maintains both stability and causality would need to include terms beyond the ones in Eq. (1). In the low-energy limit of an underlying realistic theory at the Planck scale with spontaneous Lorentz violation, any such terms would emerge as higher-dimensional nonrenormalizable operators that must be included in the standard-model extension at energies determined by the Planck scale. The structure of the

dispersion relation would correspondingly change. In a concordant frame, it would need to remain of the form (5) for small 3-momenta but would avoid spacelike 4-momenta and group velocities exceeding 1 for large 3-momenta. In fact, it can be shown by explicit construction that appropriate modifications of the dispersion relation can satisfy both the stability and causality requirements [4]. It suffices to introduce a suitable factor suppressing the coefficients for Lorentz and CPT violation only at large 3-momenta. Since the notion of large 3-momenta is frame dependent, the suppression factor itself must be frame-dependent and hence must involve Lorentz- and CPT-violating coefficients.

As an example, consider the dispersion relation (5) in the special case with only c_{00} nonzero and negative in a concordant frame. In an arbitrary frame, this dispersion relation has the form (9). To minimize complications arising from the small size of the coefficients for Lorentz and CPT violation, let the mass and the value of the Lorentz-violating coefficients be of order 1 in appropriate units. If each factor of c_{00} is combined with an exponential factor $\exp(c_{00}\lambda_0^2)$, then in an arbitrary frame the dispersion relation (9) becomes

$$(\eta_{\alpha\mu} + c_{\alpha\mu}\exp(c_{\beta\gamma}\lambda^\beta\lambda^\gamma))(\eta^\alpha{}_\nu + c^\alpha{}_\nu \exp(c_{\beta\gamma}\lambda^\beta\lambda^\gamma))\lambda^\mu\lambda^\nu - m^2 = 0. \qquad (10)$$

The presence of the exponential factors eliminates the violations of microcausality that occurred at large λ_μ in Eq. (9). The group velocity now lies below 1 for all $\vec{\lambda}$. It can also be shown that no stability problems are introduced [4].

It would evidently be interesting to identify theories in which dispersion relations of this type naturally arise. If transcendental functions of the momenta are indeed necessary to overcome the polynomial Lorentz-violating behavior in Eq. (5), a suitable lagrangian must incorporate derivative couplings of arbitrary order. It then follows that spontaneous Lorentz and CPT violation in a nonlocal theory can naturally provide the required structure for stability and causality at all scales.

The emergence of nonlocality as an important ingredient is noteworthy in part because string theories are nonlocal objects. Moreover, calculations with string field theory provided the original motivation for identifying spontaneous Lorentz and CPT violation as a potential Planck-scale signal [6] and for the development of the standard-model extension as the relevant low-energy limit [2, 3]. In fact, it can be shown that the structure of the field theory for the open bosonic string is compatible with dispersion relations of the desired type [4]. Thus, suppose in this string theory there exist nonzero Lorentz- and CPT-violating expectation values of tensor fields resulting from spontaneous symmetry breaking. Then, for example, the dispersion relation for the scalar tachyon mode contains a piece closely related to Eq. (9), but it includes also nonlocal terms of the type needed to maintain stability and causality. Although the bosonic string is merely a toy model in this context, the structural features of interest are generic to other string theories, including ones with fermions. This provides support for the existence of a stable and causal realistic fundamental theory exhibiting spontaneous Lorentz violation.

6. ACKNOWLEDGMENTS

This work is supported in part by the United States Department of Energy under grant number DE-FG02-91ER40661.

7. REFERENCES

1. For overviews of various theoretical ideas, see, for example, V.A. Kostelecký, ed., *CPT and Lorentz Symmetry,* World Scientific, Singapore, 1999.
2. V.A. Kostelecký and R. Potting, Phys. Rev. D **51** (1995) 3923.
3. D. Colladay and V.A. Kostelecký, Phys. Rev. D **55** (1997) 6760; *ibid.* **58**, 116002 (1998).
4. V.A. Kostelecký and R. Lehnert, Phys. Rev. D **63** (2001) 065008.
5. A list of particle-physics CPT tests can be found in, for example, *Review of Particle Physics,* Eur. Phys. J. C **15** (2000) 1. See also the specific experimental references below.
6. V.A. Kostelecký and S. Samuel, Phys. Rev. D **39** (1989) 683; *ibid.* **40** (1989) 1886; Phys. Rev. Lett. **63** (1989) 224; *ibid.* **66** (1991) 1811; V.A. Kostelecký and R. Potting, Nucl. Phys. B **359** (1991) 545; Phys. Lett. B **381** (1996) 89; Phys. Rev. D **63** (2001) 046007; V.A. Kostelecký, M. Perry, and R. Potting, Phys. Rev. Lett. **84** (2000) 4541.
7. V.A. Kostelecký, in B.N. Kursunoglu, S.L. Mintz, and A. Perlmutter, eds., *High-Energy Physics and Cosmology,* Plenum, New York, 1997 (hep-ph/9704264); *Physics of Mass,* Plenum, New York, 1999 (hep-ph/9810239); *Elementary Particles and Gravitation,* Plenum, New York, 1999 (hep-ph/hep-ph/0005280).
8. E731 Collaboration, L.K. Gibbons et al., Phys. Rev. D **55** (1997) 6625.
9. D. Colladay and V.A. Kostelecký, Phys. Lett. B **344** (1995) 259; Phys. Rev. D **52** (1995) 6224; V.A. Kostelecký and R. Van Kooten, Phys. Rev. D **54** (1996) 5585.
10. OPAL Collaboration, R. Ackerstaff *et al.*, Z. Phys. C **76** (1997) 401; DELPHI Collaboration, M. Feindt *et al.*, preprint DELPHI 97-98 CONF 80 (July 1997); BELLE Collaboration, K. Abe *et al.*, Phys. Rev. Lett. **86** (2001) 3228.
11. V.A. Kostelecký, Phys. Rev. Lett. **80** (1998) 1818; Phys. Rev. D **61** (2000) 016002; hep-ph/0104120.
12. KTeV Collaboration, Y.B. Hsiung *et al.*, Nucl. Phys. Proc. Suppl. **86** (2000) 312.
13. R. Bluhm, V.A. Kostelecký and N. Russell, Phys. Rev. Lett. **79** (1997) 1432; Phys. Rev. D **57** (1998) 3932.
14. G. Gabrielse *et al.*, in Ref. [1]; Phys. Rev. Lett. **82** (1999) 3198.
15. H. Dehmelt *et al.*, Phys. Rev. Lett. **83** (1999) 4694.
16. R. Mittleman, I. Ioannou, and H. Dehmelt, in Ref. [1]; R. Mittleman *et al.*, Phys. Rev. Lett. **83** (1999) 2116.
17. R. Bluhm, V.A. Kostelecký and N. Russell, Phys. Rev. Lett. **82** (1999) 2254.
18. D. Phillips *et al.*, Phys. Rev. D **63** (2001) 111101.
19. R. Bluhm, V.A. Kostelecký and C.D. Lane, Phys. Rev. Lett. **84** (2000) 1098.
20. V.W. Hughes *et al.*, presented at the Hydrogen II Conference, Tuscany, Italy, June 2000.

21. V.W. Hughes, H.G. Robinson, and V. Beltran-Lopez, Phys. Rev. Lett. **4** (1960) 342; R.W.P. Drever, Philos. Mag. **6** (1961) 683; J.D. Prestage *et al.*, Phys. Rev. Lett. **54** (1985) 2387; S.K. Lamoreaux *et al.*, Phys. Rev. Lett. **57** (1986) 3125; Phys. Rev. A **39** (1989) 1082; T.E. Chupp *et al.*, Phys. Rev. Lett. **63** (1989) 1541; C.J. Berglund *et al.*, Phys. Rev. Lett. **75** (1995) 1879.
22. V.A. Kostelecký and C.D. Lane, Phys. Rev. D **60** (1999) 116010; J. Math. Phys. **40** (1999) 6245.
23. L.R. Hunter *et al.*, in Ref. [1] and references therein.
24. D. Bear *et al.*, Phys. Rev. Lett. **85** (2000) 5038.
25. R. Bluhm and V.A. Kostelecký, Phys. Rev. Lett. **84** (2000) 1381.
26. B. Heckel *et al.*, in B.N. Kursunoglu, S.L. Mintz, and A. Perlmutter, eds., *Elementary Particles and Gravitation,* Plenum, New York, 1999.
27. S.M. Carroll, G.B. Field, and R. Jackiw, Phys. Rev. D **41** (1990) 1231.
28. R. Jackiw and V.A. Kostelecký, Phys. Rev. Lett. **82** (1999) 3572.
29. M. Pérez-Victoria, Phys. Rev. Lett. **83** (1999) 2518; J.M. Chung, Phys. Lett. B **461** (1999) 138.
30. O. Bertolami *et al.*, Phys. Lett. B **395** (1997) 178.
31. J. Rosner, Am. J. Phys. **64** (1996) 982; J. Rosner and S.A. Slezak, Am. J. Phys. **69** (1996) 44.
32. V.A. Kostelecký and A. Roberts, Phys. Rev. D **63** (2001) 096002.
33. V. Barger, S. Pakvasa, T. Weiler, and K. Whisnant, Phys. Rev. Lett. **85** (2000) 5055; T. Weiler, these proceedings.
34. D. Colladay, these proceedings.
35. W. Pauli, Phys. Rev. **58** (1940) 716.
36. C. Adam and F. Klinkhamer, hep-ph/0101087.
37. S. Coleman and S. Glashow, Phys. Rev. D **59** (1999) 116008.
38. O. Bertolami and C.S. Carvalho, Phys. Rev. D **61** (2000) 103002.